One Dimensional Collisions

Level 3 Physics

January 2013

Cart Collisions

Recall the demonstration of the two carts on the rail.

In one example, all of the velocity was transferred from the first cart to the second cart.

In the second example, the carts combined after the collision with half the initial velocity.

Cart Collisions

Recall the demonstration of the two carts on the rail.

In one example, all of the velocity was transferred from the first cart to the second cart.

In the second example, the carts combined after the collision with half the initial velocity.

These are two different types of collisions.

Energy

Collisions are classified by the energy transformations that occur.

Energy

Energy is the ability to do work.

Many types of energy exist, and *kinetic energy* (energy due to motion) and *potential energy* (energy due to position) are the two major categories.

Energy

Collisions are classified by the energy transformations that occur.

Energy

Energy is the ability to do work.

Many types of energy exist, and *kinetic energy* (energy due to motion) and *potential energy* (energy due to position) are the two major categories.

Law of Conservation of Energy

Energy can neither be created nor destroyed but can be converted from one form to another.

Kinetic Energy

Kinetic Energy

The kinetic energy K of an object is the energy due to its motion. An object of mass m traveling with velocity v has kinetic energy

$$K = \frac{1}{2}mv^2$$

Kinetic Energy

Kinetic Energy

The kinetic energy K of an object is the energy due to its motion. An object of mass m traveling with velocity v has kinetic energy

$$K = \frac{1}{2}mv^2$$

Notice that kinetic energy is always non-negative and the units of kinetic energy are Joules (J), where $J = N * m = \frac{kg * m^2}{s^2}$.

Kinetic Energy

Kinetic Energy

The kinetic energy K of an object is the energy due to its motion. An object of mass m traveling with velocity v has kinetic energy

$$K = \frac{1}{2}mv^2$$

Notice that kinetic energy is always non-negative and the units of kinetic energy are Joules (J), where $J = N * m = \frac{kg*m^2}{s^2}$.

The kinetic energy of a system is the sum of the kinetic energies of each component.

Collision Types

Collisions are classified by the relationship between the initial and final kinetic energies (K_i and K_f, respectively) of the system.

Collision Types

Collisions are classified by the relationship between the initial and final kinetic energies (K_i and K_f, respectively) of the system.

- **Elastic Collision**: $K_i = K_f$ (no change)
- **Inelastic Collision**: $K_i > K_f$ (kinetic energy lost)
- **Superelastic Collision**: $K_i < K_f$ (kinetic energy gained)

Elastic Collisions

In an elastic collision, the kinetic energy stays constant ($K_i = K_f$), producing the following equations for a system with two objects:

- Conservation of Momentum:
  $m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$
- Conservation of Kinetic Energy:
  $\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$

Elastic Collisions

In an elastic collision, the kinetic energy stays constant ($K_i = K_f$), producing the following equations for a system with two objects:

- Conservation of Momentum:
  $m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$
- Conservation of Kinetic Energy:
  $\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$

Elastic collisions only actually occur on the microscopic level between atoms. However, they are often used to model billiard ball collisions at the macroscopic level.

Elastic Collisions Velocity

A simple relationship between the initial and final velocities of each object can be derived for elastic collisions from the two equations.

Elastic Collisions Velocity

A simple relationship between the initial and final velocities of each object can be derived for elastic collisions from the two equations.

Factoring the conservation of kinetic energy formula gives

$$\frac{1}{2}m_1 * (v_{1f}^2 - v_{1i}^2) = \frac{1}{2}m_2 * (v_{2i}^2 - v_{2f}^2)$$

$$m_1(v_{1f} + v_{1i})(v_{1f} - v_{1i}) = m_2(v_{2i} + v_{2f})(v_{2i} - v_{2f})$$

Elastic Collisions Velocity

A simple relationship between the initial and final velocities of each object can be derived for elastic collisions from the two equations.

Factoring the conservation of kinetic energy formula gives

$$\frac{1}{2}m_1 * (v_{1f}^2 - v_{1i}^2) = \frac{1}{2}m_2 * (v_{2i}^2 - v_{2f}^2)$$
$$m_1(v_{1f} + v_{1i})(v_{1f} - v_{1i}) = m_2(v_{2i} + v_{2f})(v_{2i} - v_{2f})$$

Also, rewriting the conservation of momentum formula gives

$$m_1(v_{1f} - v_{1i}) = m_2(v_{2i} - v_{2f})$$

Elastic Collisions Velocity, cont'd.

Dividing these two intermediate results gives

$$\frac{m_1(v_{1f}+v_{1i})(v_{1f}-v_{1i})}{m_1(v_{1f}-v_{1i})}=\frac{m_2(v_{2i}+v_{2f})(v_{2i}-v_{2f})}{m_2(v_{2i}-v_{2f})}$$

$$v_{1i}+v_{1f}=v_{2i}+v_{2f}$$

Elastic Collisions Velocity, cont'd.

Dividing these two intermediate results gives

$$\frac{m_1(v_{1f} + v_{1i})(v_{1f} - v_{1i})}{m_1(v_{1f} - v_{1i})} = \frac{m_2(v_{2i} + v_{2f})(v_{2i} - v_{2f})}{m_2(v_{2i} - v_{2f})}$$

$$v_{1i} + v_{1f} = v_{2i} + v_{2f}$$

Elastic Collisions

The initial and final velocities for an elastic collision between two objects (not necessarily of equal mass) are related by

$$v_{1i} + v_{1f} = v_{2i} + v_{2f}$$

Note that this equation is independent of the objects' mass.

Inelastic Collisions

In an inelastic collision, kinetic energy is lost through conversion to other energy forms, so $K_i > K_f$.

Inelastic Collisions

In an inelastic collision, kinetic energy is lost through conversion to other energy forms, so $K_i > K_f$.

All collisions on the macroscopic level are inelastic to various degrees, meaning that varying amounts of kinetic energy are lost.

Inelastic Collisions

In an inelastic collision, kinetic energy is lost through conversion to other energy forms, so $K_i > K_f$.

All collisions on the macroscopic level are inelastic to various degrees, meaning that varying amounts of kinetic energy are lost.

The special type of inelastic collision where the kinetic energy lost is maximized is called a **perfectly inelastic collision**.

Perfectly Inelastic Collision Velocity

Using calculus, it can be shown that to maximize the kinetic energy lost, the two objects must stick together after the collision.

This condition allows the conservation of momentum equation to be rewritten with $v_f = v_{1f} = v_{2f}$ as

$$m_1 v_{1i} + m_2 v_{2i} = v_f (m_1 + m_2)$$

Perfectly Inelastic Collision Velocity

Using calculus, it can be shown that to maximize the kinetic energy lost, the two objects must stick together after the collision.

This condition allows the conservation of momentum equation to be rewritten with $v_f = v_{1f} = v_{2f}$ as

$$m_1 v_{1i} + m_2 v_{2i} = v_f (m_1 + m_2)$$

Perfectly Inelastic Collisions

The final shared velocity of two objects in a perfectly inelastic collision is

$$v_{1f} = v_{2f} = v_f = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2}$$

Superelastic Collisions

In a superelastic collision, kinetic energy is gained through conversion from other forms, so $K_i < K_f$.

Superelastic Collisions

In a superelastic collision, kinetic energy is gained through conversion from other forms, so $K_i < K_f$.

Explosions such as fireworks are a form of superelastic collision.

Momentum Conservation

There are two cars, A and B, that have initial momentum 20 $\frac{kg*m}{s}$ and -14 $\frac{kg*m}{s}$, respectively. The final momentum of car A is 2 $\frac{kg*m}{s}$. What is the final momentum of car B?

1. 4 $\frac{kg*m}{s}$
2. 8 $\frac{kg*m}{s}$
3. 32 $\frac{kg*m}{s}$
4. 36 $\frac{kg*m}{s}$

Momentum Conservation

There are two cars, A and B, that have initial momentum 20 $\frac{kg*m}{s}$ and -14 $\frac{kg*m}{s}$, respectively. The final momentum of car A is 2 $\frac{kg*m}{s}$. What is the final momentum of car B?

1. 4 $\frac{kg*m}{s}$
2. 8 $\frac{kg*m}{s}$
3. 32 $\frac{kg*m}{s}$
4. 36 $\frac{kg*m}{s}$

Correct Answer: 1
(The law of conservation of momentum gives $p_{1i} + p_{2i} = p_{1f} + p_{2f}$.)

Transfer of Velocity

Recall the demonstration of the collision between two carts where all of the velocity was transferred from the first cart to the second cart. What type of collision is this?

1. Elastic collision
2. Inelastic collision
3. Superelastic collision

Transfer of Velocity

Recall the demonstration of the collision between two carts where all of the velocity was transferred from the first cart to the second cart. What type of collision is this?

1. Elastic collision
2. Inelastic collision
3. Superelastic collision

Correct Answer: 1
(The kinetic energy before and after the collision is the same.)

Sharing Final Velocity

Recall the demonstration of the collision between two carts where the carts combined after the collision with half the velocity. How much kinetic energy is lost if each cart has a mass m and the initial velocity of the first cart was v?

1. $-\frac{1}{4}mv^2$
2. 0
3. $\frac{1}{4}mv^2$
4. $\frac{3}{4}mv^2$

Sharing Final Velocity

Recall the demonstration of the collision between two carts where the carts combined after the collision with half the velocity. How much kinetic energy is lost if each cart has a mass m and the initial velocity of the first cart was v?

1. $-\frac{1}{4}mv^2$
2. 0
3. $\frac{1}{4}mv^2$
4. $\frac{3}{4}mv^2$

Correct Answer: 3
(The kinetic energy lost in this perfectly inelastic collision is $K_i - K_f = \frac{1}{2}mv^2 - 2(\frac{1}{2}m(\frac{v}{2})^2) = \frac{1}{4}mv^2$.)

Elastic Collision

Suppose two carts start at opposite sides of the rail and move towards each other. If each cart has a mass m, a speed v initially, and the collision is elastic, what is the behavior of the system after the collision?

1. The carts continue to the right with speed $\frac{v}{2}$
2. The carts stop at the center of the rail
3. The carts bounce off each other and both reverse their direction while maintaining a speed v

Elastic Collision

Suppose two carts start at opposite sides of the rail and move towards each other. If each cart has a mass m, a speed v initially, and the collision is elastic, what is the behavior of the system after the collision?

1. The carts continue to the right with speed $\frac{v}{2}$
2. The carts stop at the center of the rail
3. The carts bounce off each other and both reverse their direction while maintaining a speed v

Correct Answer: 3
(The total momentum is 0, so the final velocities of the carts must be opposites. The kinetic energy was positive initially so must also be positive after an elastic collision.)

Perfectly Inelastic Collision

Suppose two carts start at opposite sides of the rail and move towards each other. If each cart has a mass m, a speed v initially, and the collision is perfectly inelastic, what is the behavior of the system after the collision?

1. The carts continue to the right with speed $\frac{v}{2}$
2. The carts stop at the center of the rail
3. The carts bounce off each other and both reverse their direction while maintaining a speed v

Perfectly Inelastic Collision

Suppose two carts start at opposite sides of the rail and move towards each other. If each cart has a mass m, a speed v initially, and the collision is perfectly inelastic, what is the behavior of the system after the collision?

1. The carts continue to the right with speed $\frac{v}{2}$
2. The carts stop at the center of the rail
3. The carts bounce off each other and both reverse their direction while maintaining a speed v

Correct Answer: 2
(The total momentum is 0, so the final velocities of the carts must be opposites. Perfectly inelastic collisions lose the maximum possible kinetic energy, which occurs in this situation when the cars no longer move following the collision.)

Newton's Cradle

Newton's Cradle can be approximated using the theory of elastic collisions. Suppose that when two balls were set in motion on a Newton's Cradle with velocity v, the remaining three balls were displaced with a velocity $\frac{2}{3}v$. Why is this impossible?

1. Violates conservation of momentum
2. Violates conservation of kinetic energy
3. Violates both conservation of momentum and kinetic energy

Newton's Cradle

Newton's Cradle can be approximated using the theory of elastic collisions. Suppose that when two balls were set in motion on a Newton's Cradle with velocity v, the remaining three balls were displaced with a velocity $\frac{2}{3}v$. Why is this impossible?

1. Violates conservation of momentum
2. Violates conservation of kinetic energy
3. Violates both conservation of momentum and kinetic energy

Correct Answer: 2
(Conservation of momentum is true since $2 * v = 3 * (\frac{2}{3}v)$, but conservation of kinetic energy is not true since $2(\frac{1}{2}mv^2) \neq 3(\frac{1}{2}m(\frac{2}{3}v)^2)$.)