

1. Trovare un'equazione cartesiana per la retta

$$r: \begin{cases} x = 1 - t \\ y = -\frac{1}{2} + 2t \end{cases}$$

Soluzione.

$$\begin{cases} t = 1 - x \\ y = -\frac{1}{2} + 2(1 - x) \end{cases} \quad \begin{cases} \text{"} \\ y = -\frac{1}{2} + 2 - 2x \end{cases}$$

Un'equazione cartesiana di r è:

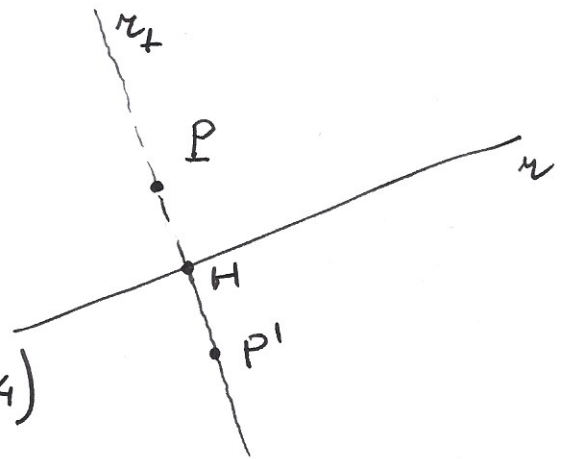
$$4x + 2y - 3 = 0$$

2. Trovare le coordinate del p.to P' , simmetrico di $P = (\frac{1}{2}, 4)$ rispetto alla retta $r: y = \frac{1}{3}x - \frac{7}{6}$

Soluzione

- Retta perpendicolare a r e passante per P :

$$r_{\perp}: \begin{cases} x = \frac{1}{2} + \frac{1}{3}t \\ y = 4 - t \end{cases}$$



- $H = r \cap r_{\perp} = (2, -\frac{1}{2})$

- $P' = 2H - P = 2(2, -\frac{1}{2}) - (\frac{1}{2}, 4)$
 $= (4, -1) - (\frac{1}{2}, 4)$
 $= (\frac{7}{2}, -5)$.

3. Trovare le coordinate dell'ortocentro del triangolo avente per lati le rette

$$r_1: \begin{cases} x = t \\ y = 4 + 2t \end{cases}$$

$$r_2: \begin{cases} x = u \\ y = 4 - 2u \end{cases}$$

$$r_3: \begin{cases} x = s \\ y = -4 \end{cases}$$

$$A = r_1 \cap r_2: \begin{cases} t = u \\ 4 + 2t = 4 - 2u \end{cases} \quad \begin{cases} t = u \\ 2u = -2u \end{cases} \quad \begin{cases} t = u \\ 4u = 0 \end{cases} \quad \begin{cases} u = 0 \end{cases}$$

$$A = (0, 4)$$

$$B = r_2 \cap r_3: \begin{cases} u = s \\ 4 - 2u = -4 \end{cases} \quad \begin{cases} u = s \\ u = \frac{8}{2} = 4 \end{cases} \quad B = (4, -4)$$

$$C = r_1 \cap r_3: \begin{cases} t = s \\ 4 + 2t = -4 \end{cases} \quad \begin{cases} t = s \\ t = -4 \end{cases} \quad C = (-4, -4)$$

$$m_{AB} = \frac{-4 - 4}{4 - 0} = -2$$

Equazione dell'altessa
relativa a AB:

$$y + 4 = \frac{1}{2}(x + 4)$$

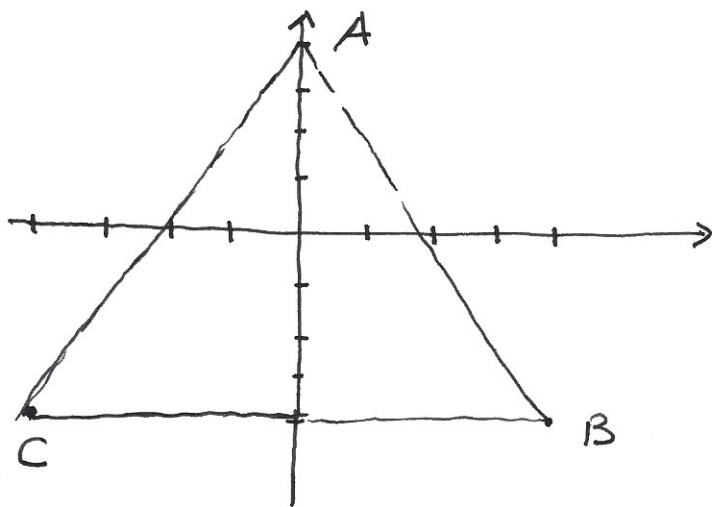
$$h_1: y = \frac{1}{2}x - 2$$

Equazione dell'altessa relativa
a BC:

$$h_2: x = 0$$

$$h_1 \cap h_2 = \begin{cases} x = 0 \\ y = \frac{1}{2} \cdot 0 - 2 \end{cases} \quad \begin{cases} x = 0 \\ y = -2 \end{cases}$$

$$\text{Ortocentro} = (0, -2)$$



4. Determinare la distanza tra le rette di equat. parametriche

$$r: \begin{cases} x = t \\ y = t \end{cases} \quad e \quad s: \begin{cases} x = u \\ y = 2 + u \end{cases}$$

$$r: y = x \rightarrow x - y = 0$$

$$s: y = 2 + x \rightarrow x - y + 2 = 0$$

$$\text{distanza} = \frac{|0 - 0 + 2|}{\sqrt{1 + 1}} = \frac{2}{\sqrt{2}} = \sqrt{2}$$

5. Siano $A = (-2, 0)$ e $B = (2, 0)$ i vertici del triangolo ABC avente per baricentro $G = (\frac{1}{3}, \frac{4}{3})$

Determinare

(a) il vertice C

(b) un p.to P , sull'asse y , in modo tale che i triangoli APB e BPC siano equivalenti.

1 p.to (a) $G = \frac{1}{3}(A+B+C) \rightarrow C = 3G - A - B = (1, 4)$

2 p.ti (b) $P(0, t)$ $a(\triangle ABC) = \frac{4}{2}|t|$

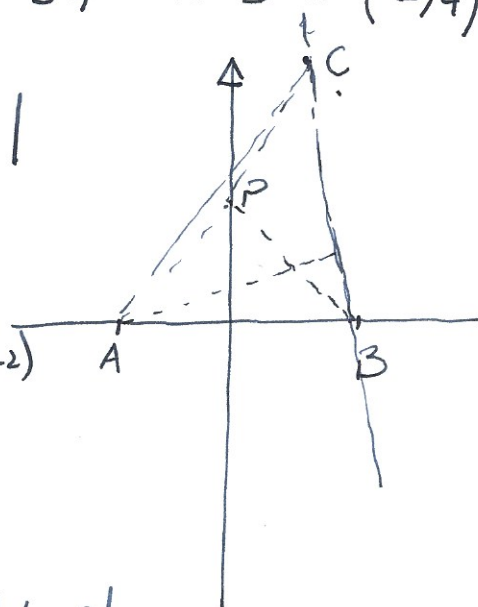
$$BC = \sqrt{(-1)^2 + 4^2} = \sqrt{17}$$

$$m_{BC} = \frac{4-0}{-1-2} = \frac{4}{-3} = -\frac{4}{3} \quad \text{retta } BC: y-0 = -\frac{4}{3}(x-2)$$

$$\text{retta } BC: 4x + y - 8 = 0 \quad (r)$$

$$h = d(P, r) = \frac{|4 \cdot 0 + t - 8|}{\sqrt{17}}$$

$$\frac{2}{2} \frac{4}{2} |t| = \frac{1}{2} \frac{\sqrt{17}}{\sqrt{17}} \frac{|t-8|}{\sqrt{17}} \Leftrightarrow 4|t| = |t-8|$$



$$1^{\circ} \text{ CASO} \quad t \geq 8$$

$$4t = t - 8 \quad \leftrightarrow \quad 3t = -8 \quad \leftrightarrow \quad t = -\frac{8}{3} \quad \text{NON ACCETTABILE}$$

$$2^{\circ} \text{ CASO} \quad 0 \leq t < 8$$

$$4t = -t + 8 \quad \leftrightarrow \quad 5t = 8 \quad \leftrightarrow \quad t = \frac{8}{5}$$

$$3^{\circ} \text{ CASO}$$

$$-4t = -t + 8 \quad \leftrightarrow \quad -3t = 8 \quad \leftrightarrow \quad t = -\frac{8}{3}$$